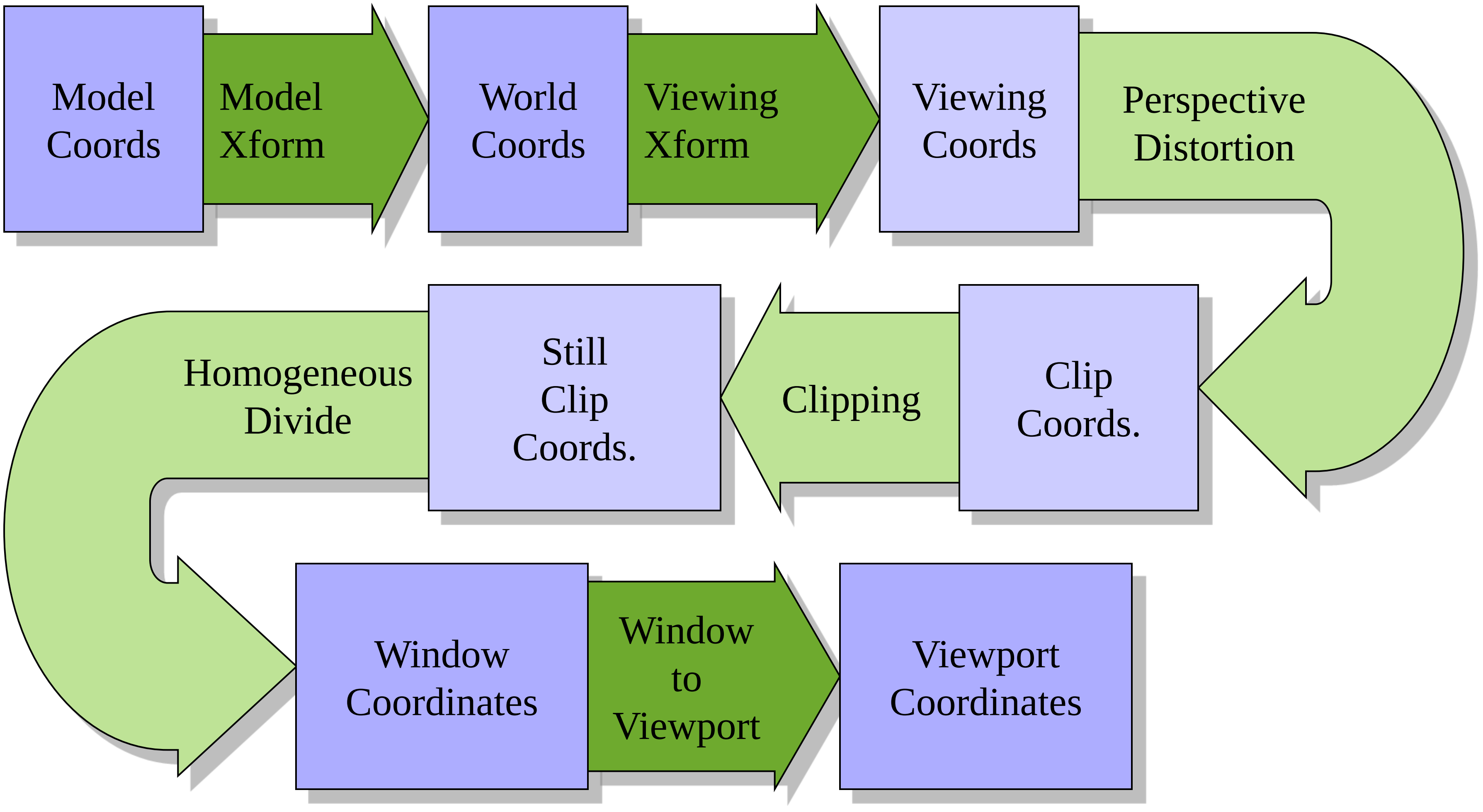
Perspective

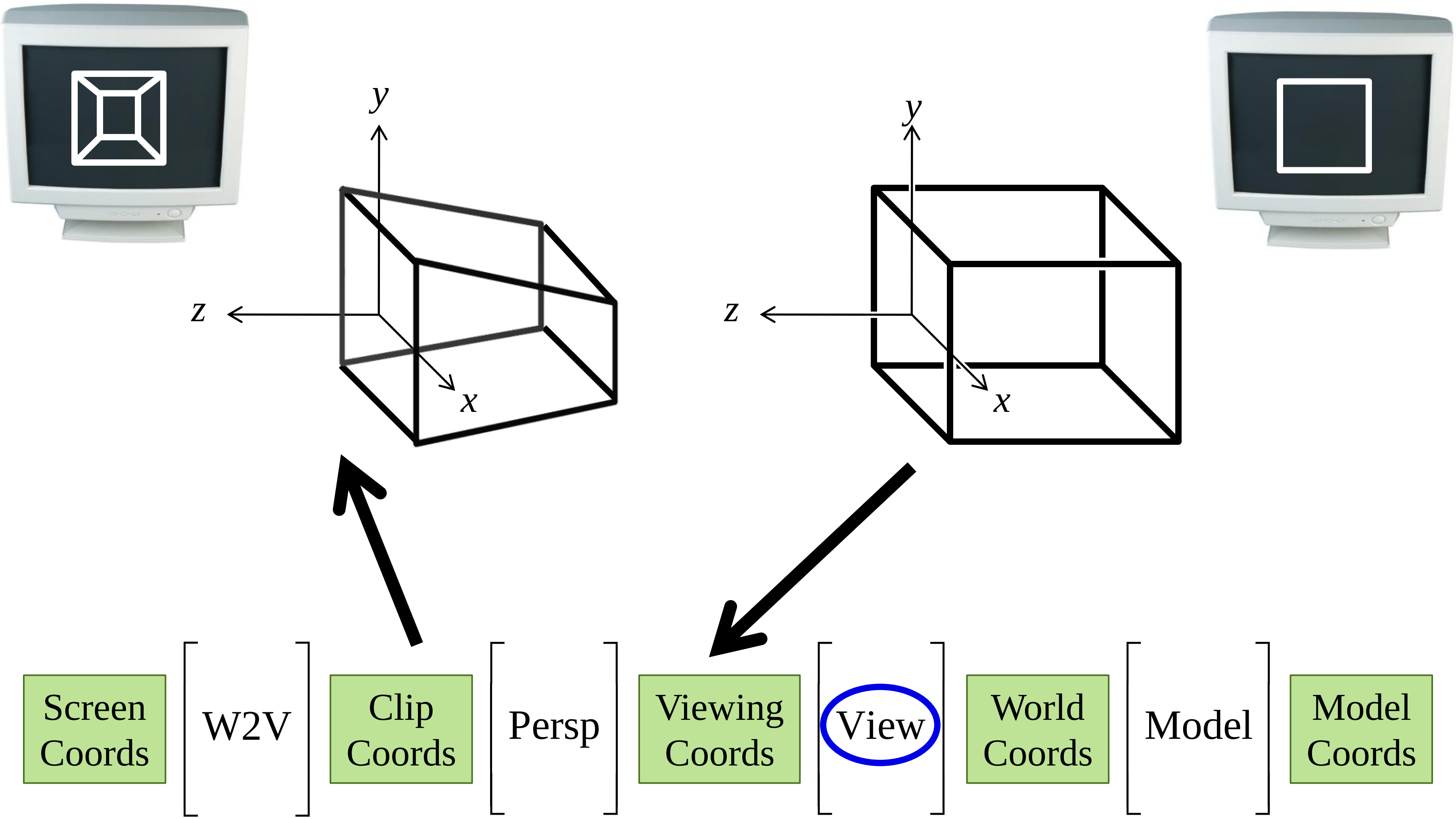
CS418 Computer Graphics

John C. Hart

Graphics Pipeline



Graphics Pipeline

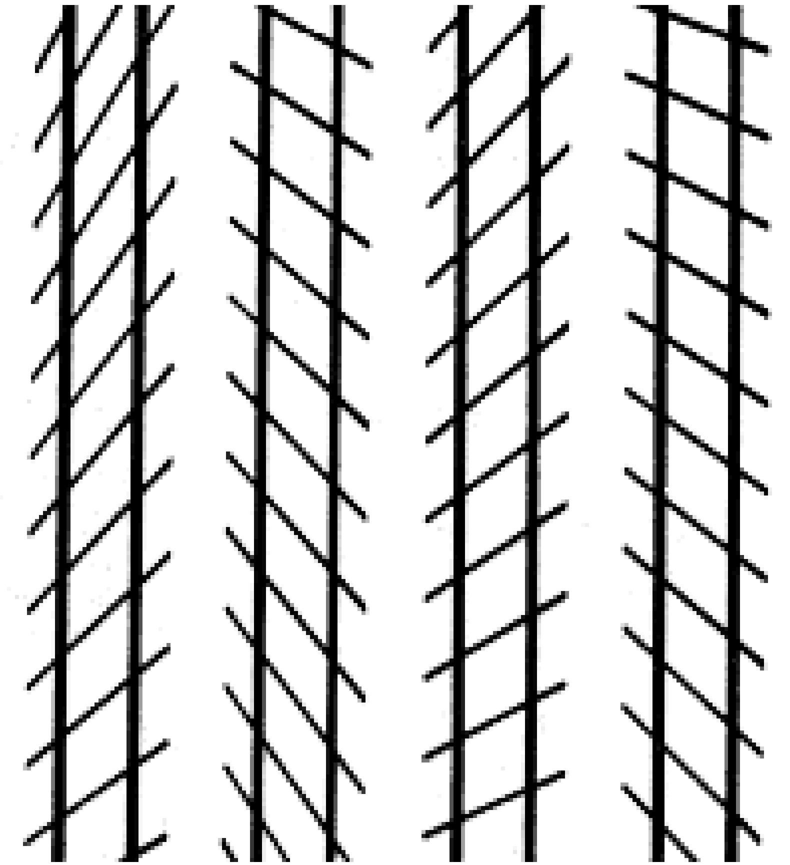


Foreshortening

Projections squash receding surfaces

Andrea Mantegna
The Lamentation over the Dead Christ

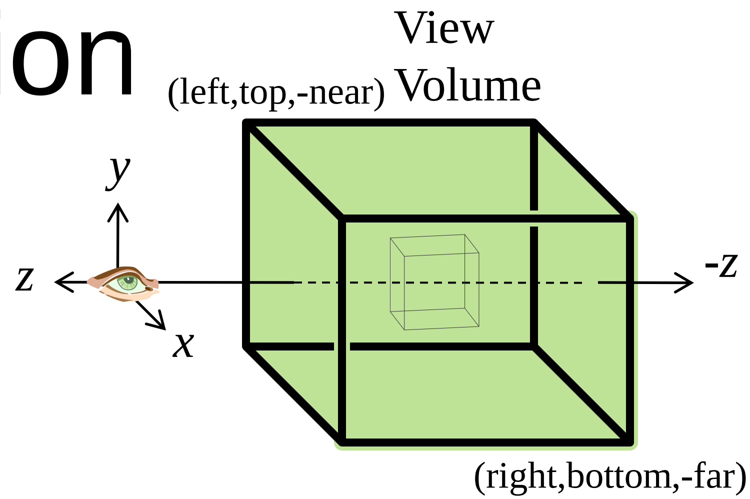
Zollner Illusion



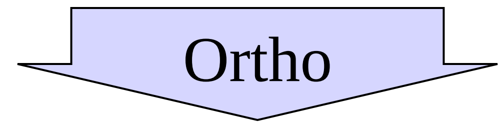
Orthographic Projection

glOrtho(left,right,bottom,top,near,far)

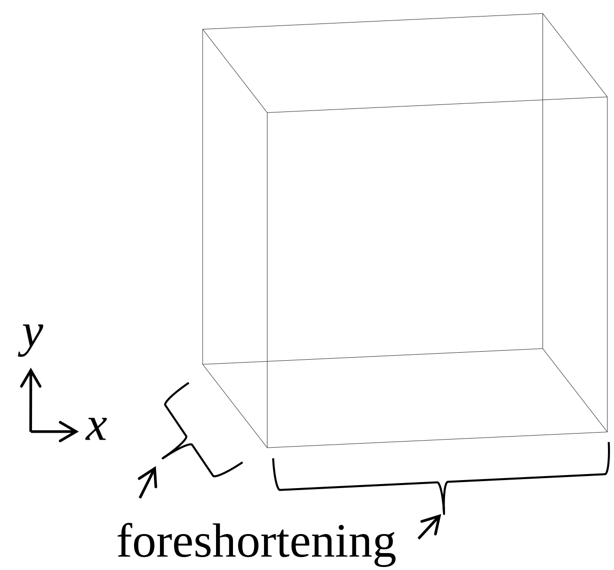
- Foreshortens
- No change in size by depth
- Projection matrix simply zeros the z- coordinate



Viewing Coordinates



Clip Coordinates

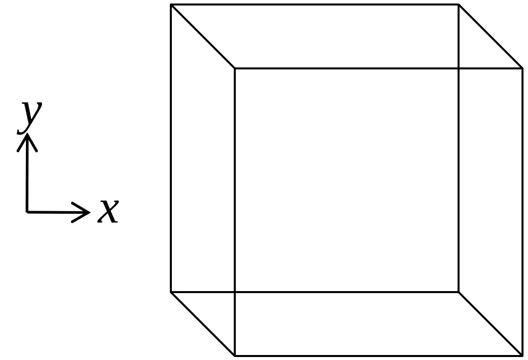


$$\begin{bmatrix} \text{W2V} \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 0 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} \text{View} \end{bmatrix} \begin{bmatrix} \text{Model} \end{bmatrix}$$

Isometric Projection

- Foreshortens by using *z*-coord to shear *x* and *y* coordinates

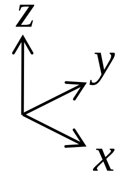
$$\begin{bmatrix} x_{clip} \\ y_{clip} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & 1 & \\ & 1 & -1 & \\ & & 0 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x_{view} \\ y_{view} \\ z_{view} \\ 1 \end{bmatrix}$$



$$x_{clip} = x_{view} + z_{view}$$
$$y_{clip} = y_{view} - z_{view}$$

- Used in videogames to place sprites

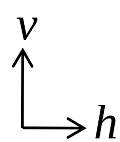
$$\begin{bmatrix} x_{clip} \\ y_{clip} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & & \\ -1 & 1 & 2 & \\ & & 0 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x_{view} \\ y_{view} \\ z_{view} \\ 1 \end{bmatrix}$$



$$x_{clip} = x_{view} + y_{view}$$
$$y_{clip} = -x_{view} + y_{view} + 2z_{view}$$

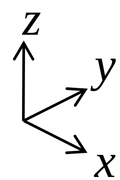
Isometric Projection

$$\begin{bmatrix} x_{\text{clip}} \\ y_{\text{clip}} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & & \\ -1 & 1 & 2 & \\ & & 0 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x_{\text{view}} \\ y_{\text{view}} \\ z_{\text{view}} \\ 1 \end{bmatrix}$$



$h = x + y$

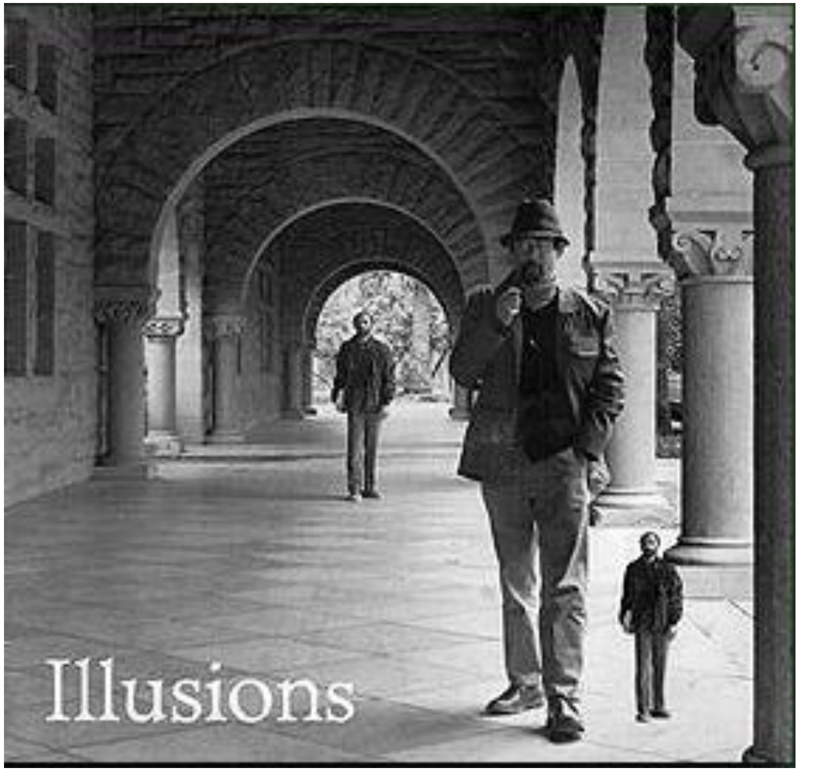
$v = -x + y + 2z$



$$\begin{bmatrix} h \\ v \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.866 & 0.866 & & \\ -0.5 & 0.5 & 1 & \\ & & 0 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective

- Brain depends on shape constancy
  - Real world objects do not resize
  - Change in size due to depth
- Closer objects larger
- Farther objects smaller



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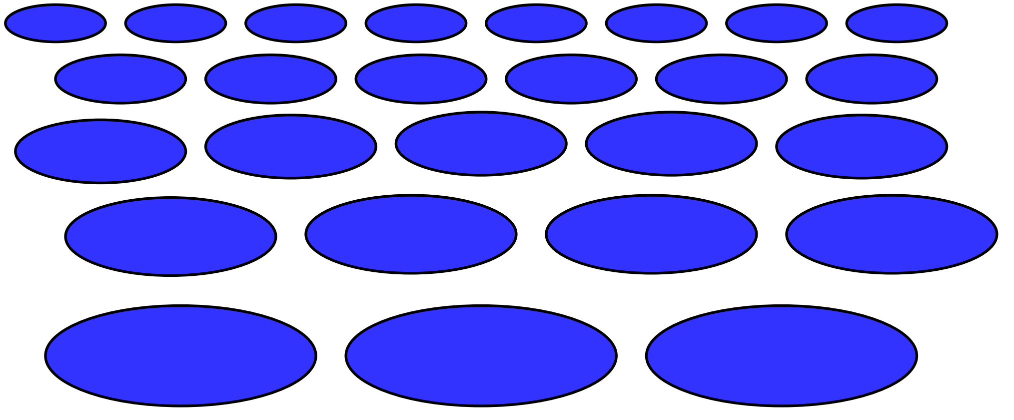
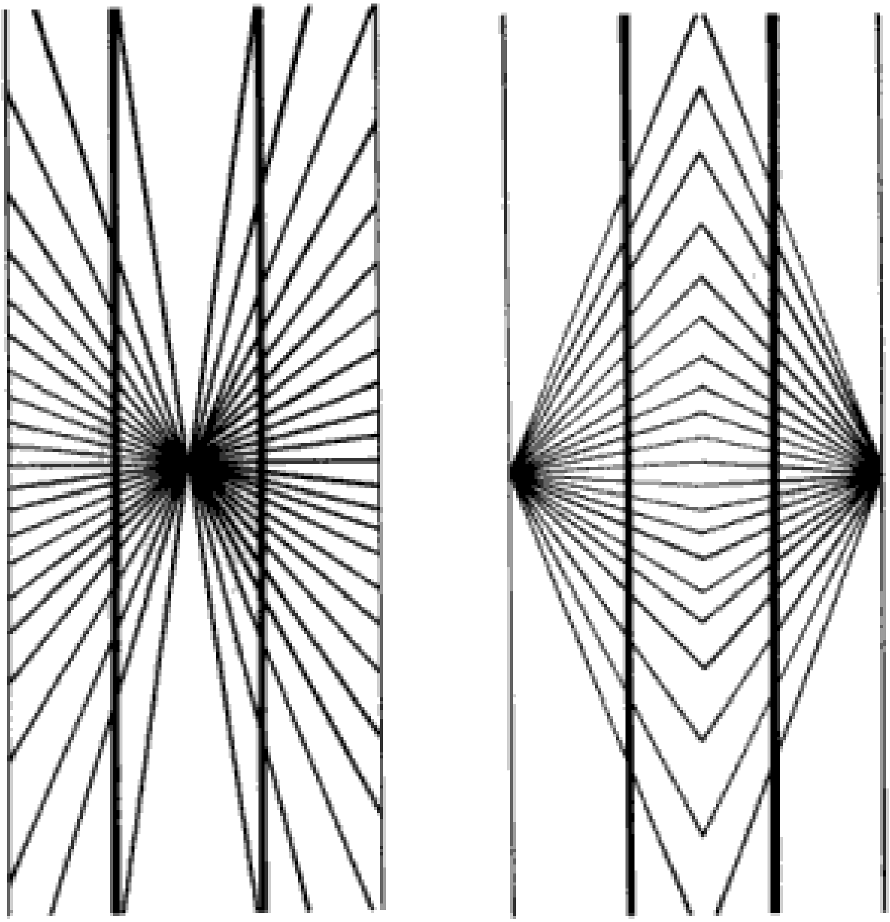
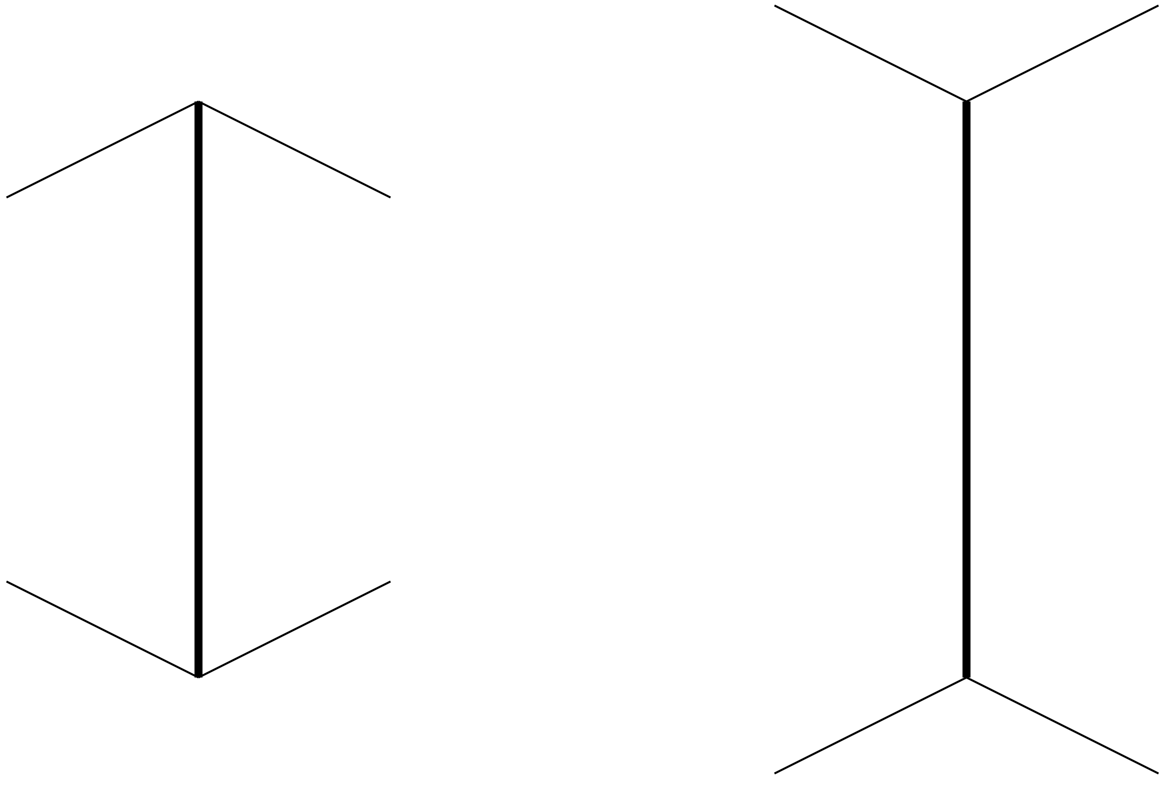
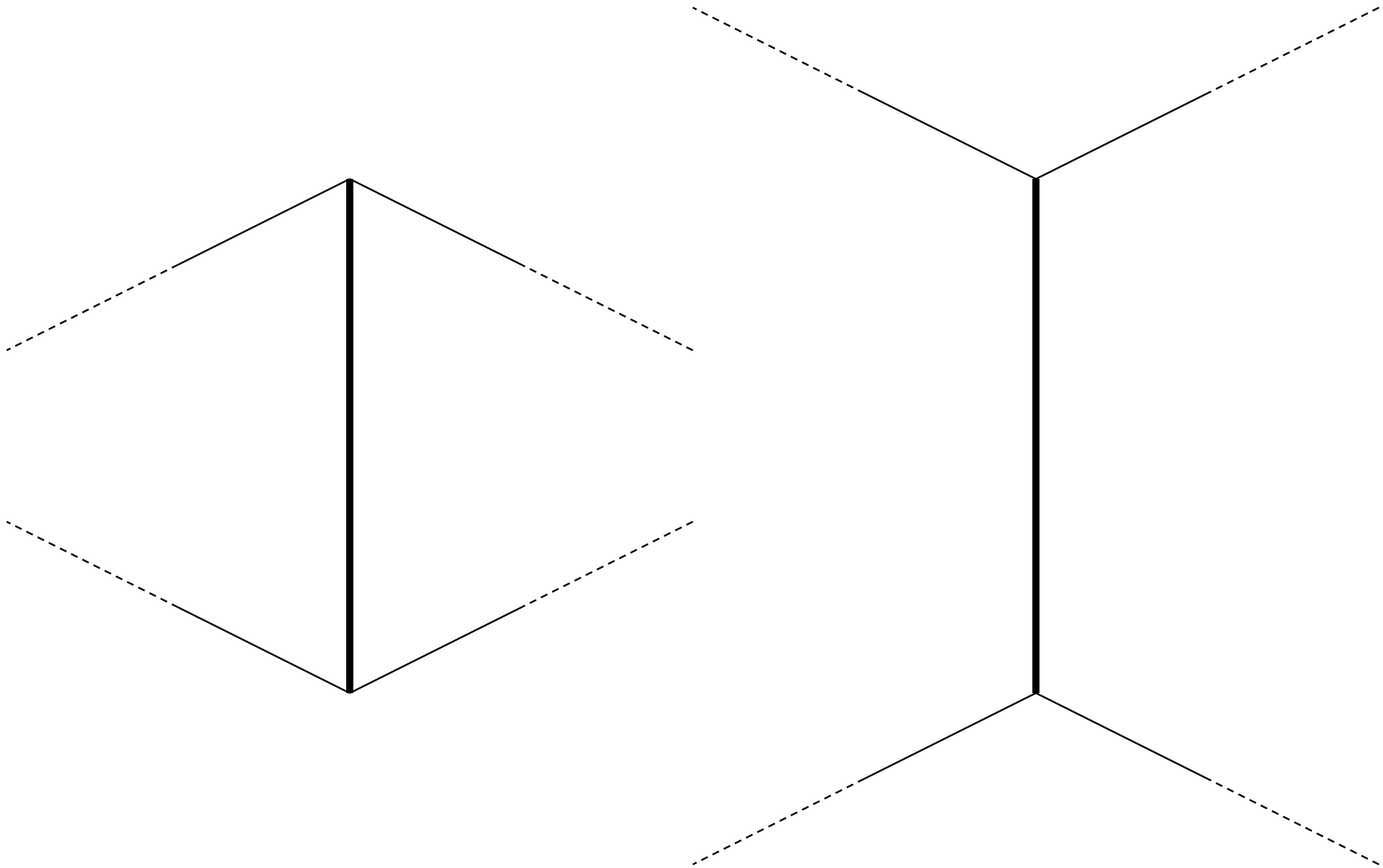


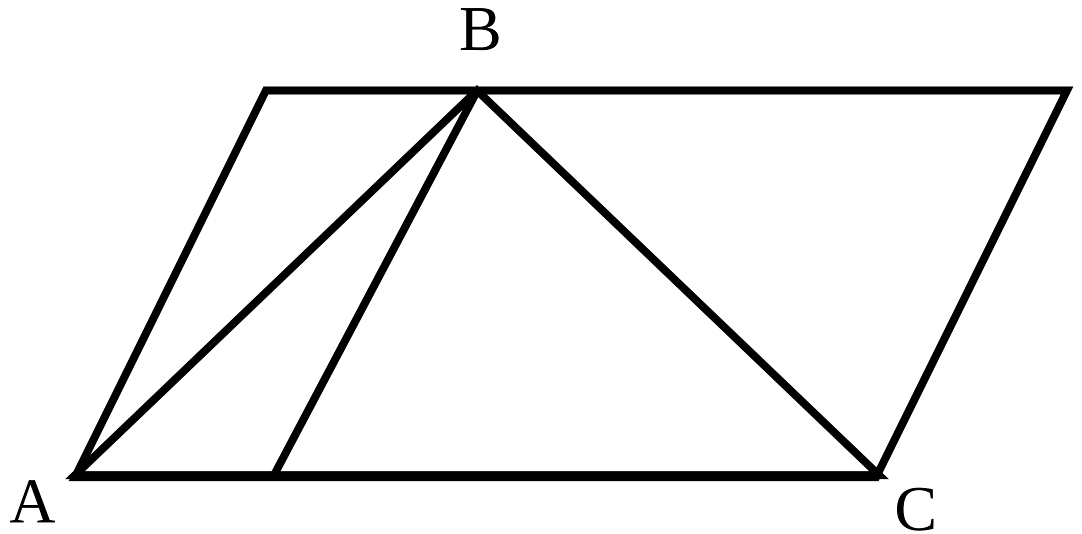
Ames Distorting Room

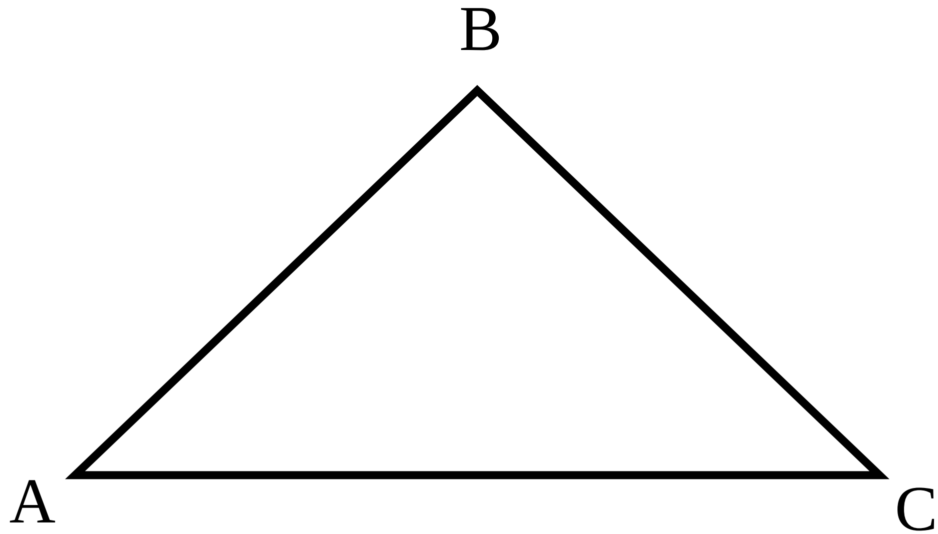
Hering Illusion





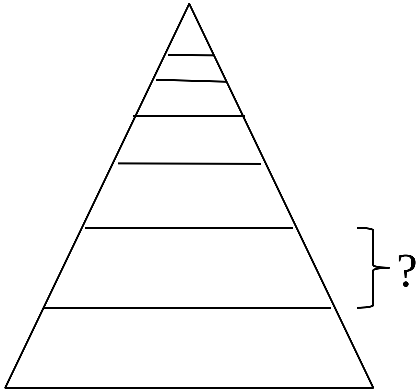



B
A
C

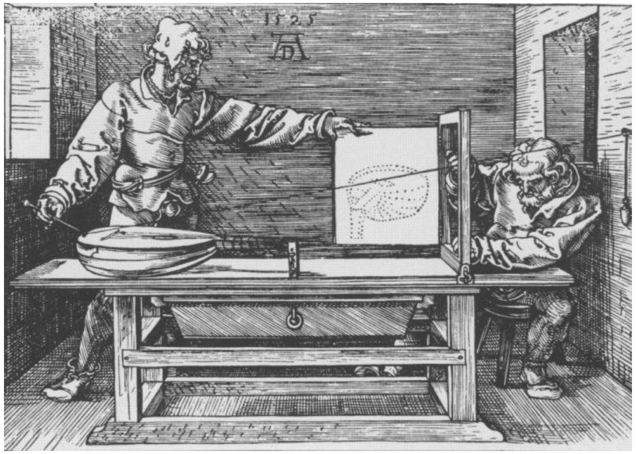

B
A
C

Linear Perspective

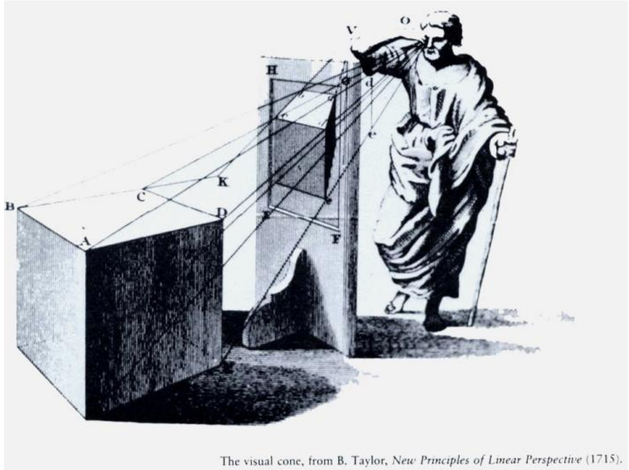
- Brain depends on shape constancy
  - Real world objects do not resize
  - Change in size due to depth
- Closer objects larger
- Farther objects smaller
- How large, how small?



Albrecht Durer woodcut c. 1525,
swiped from Marc Levoy's CS48N notes c. 2007

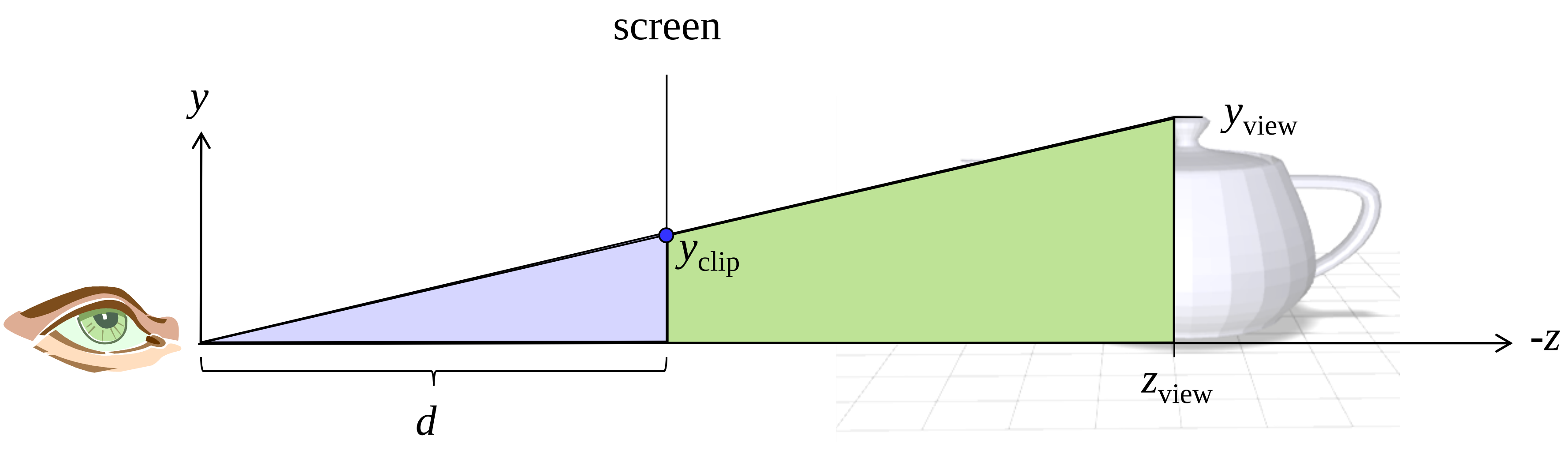


More Durer, swiped from Fredo Durand's Art of Depiction



The visual cone, from B. Taylor, *New Principles of Linear Perspective* (1715).

Linear Perspective



$$\frac{y_{\text{clip}}}{d} = \frac{y_{\text{view}}}{-z_{\text{view}}}$$

$$y_{\text{clip}} = d\,\frac{y_{\text{view}}}{-z_{\text{view}}} = \frac{y_{\text{view}}}{-z_{\text{view}}\,/\,d}$$

Homogeneous Coordinates

- Fourth homogeneous coordinate can be any non-zero value
- To find the point it corresponds to:
  - multiply all four coordinates by the same value
  - precisely the value that makes the fourth coordinate one

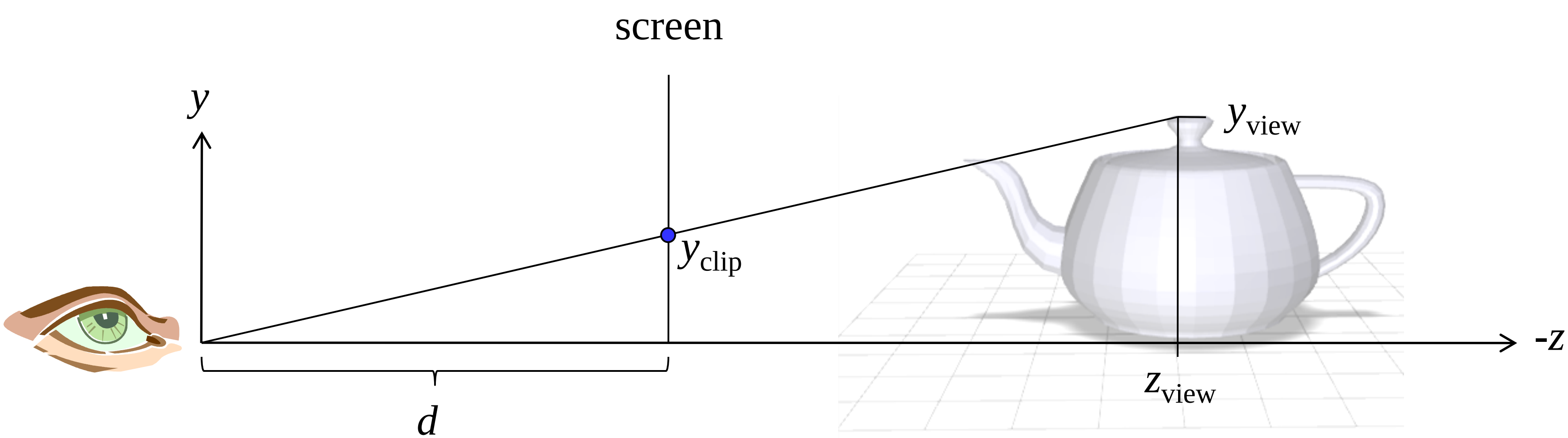
$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \equiv \begin{bmatrix} x/w \\ y/w \\ z/w \\ 1 \end{bmatrix}$$

- When homogeneous coordinate is zero
  - Denotes a “point” at infinity
  - Represents a vector instead of a point
  - Not affected by translation

$$\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

Linear Perspective



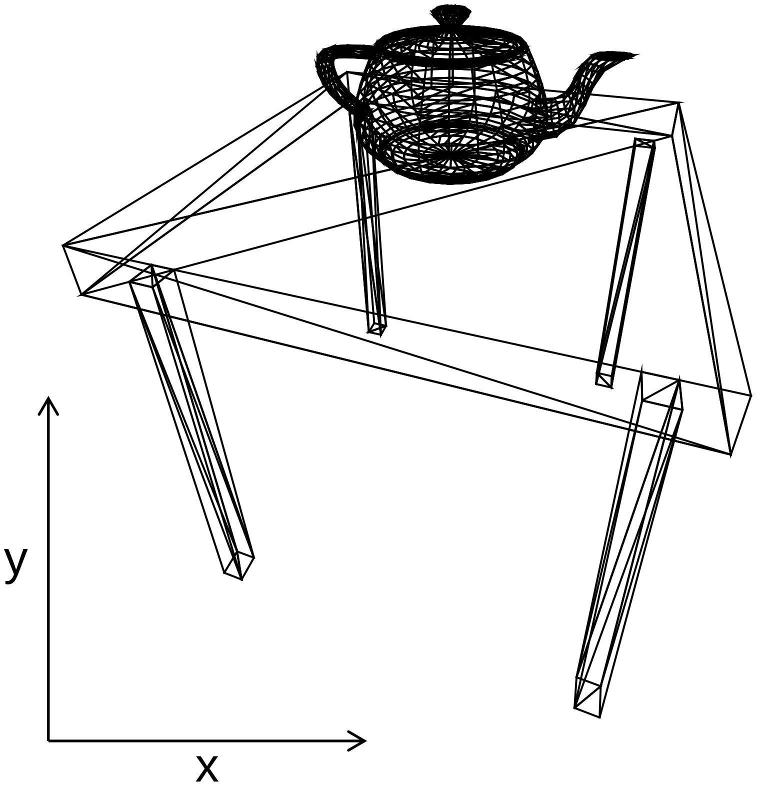
$$\frac{y_{\text{clip}}}{d} = \frac{y_{\text{view}}}{-z_{\text{view}}}$$

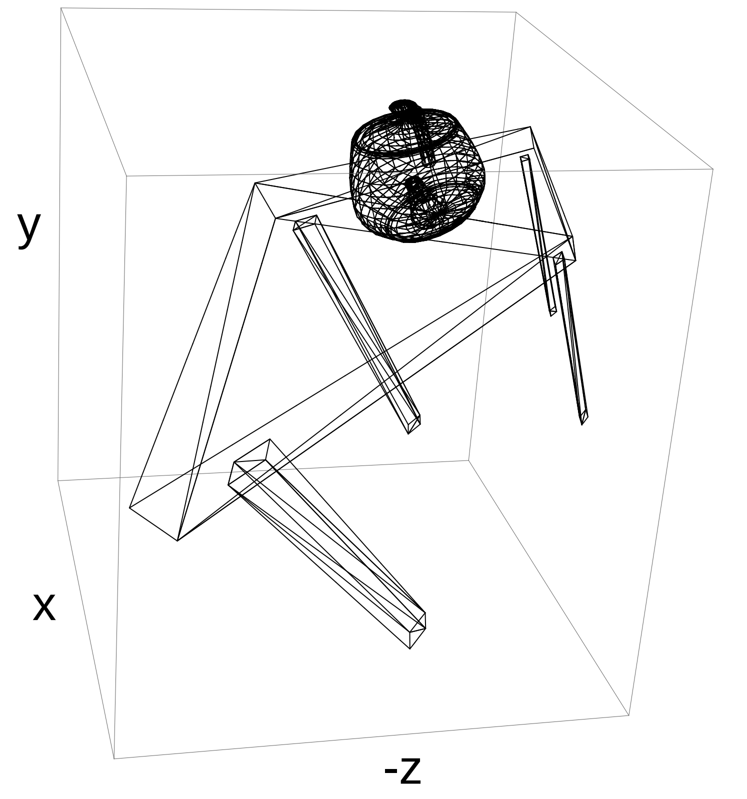
$$y_{\text{clip}} = \frac{y_{\text{view}}}{-z_{\text{view}}/d}$$

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & -1/d & 0 \end{bmatrix} \begin{bmatrix} x_{\text{view}} \\ y_{\text{view}} \\ z_{\text{view}} \\ 1 \end{bmatrix} = \begin{bmatrix} x_{\text{view}} \\ y_{\text{view}} \\ z_{\text{view}} \\ -z_{\text{view}}/d \end{bmatrix} \equiv \begin{bmatrix} \dfrac{x_{\text{view}}}{-z_{\text{view}}/d} \\ \dfrac{y_{\text{view}}}{-z_{\text{view}}/d} \\ -d \\ 1 \end{bmatrix}$$

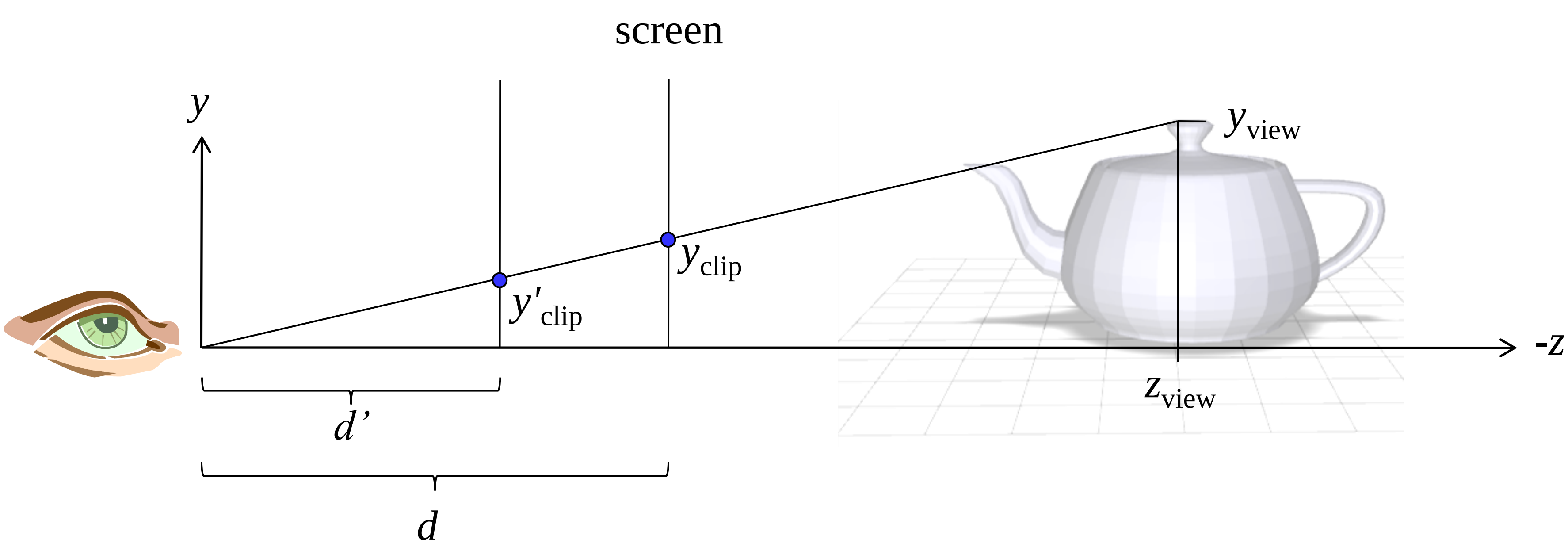
Perspective Distortion

(Using a later version of Perspective matrix that preserves depth ordering)





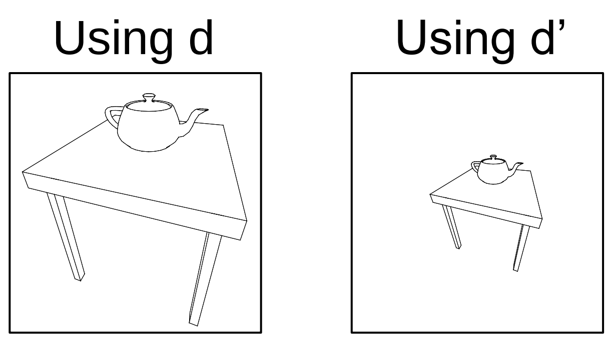
Parameter d



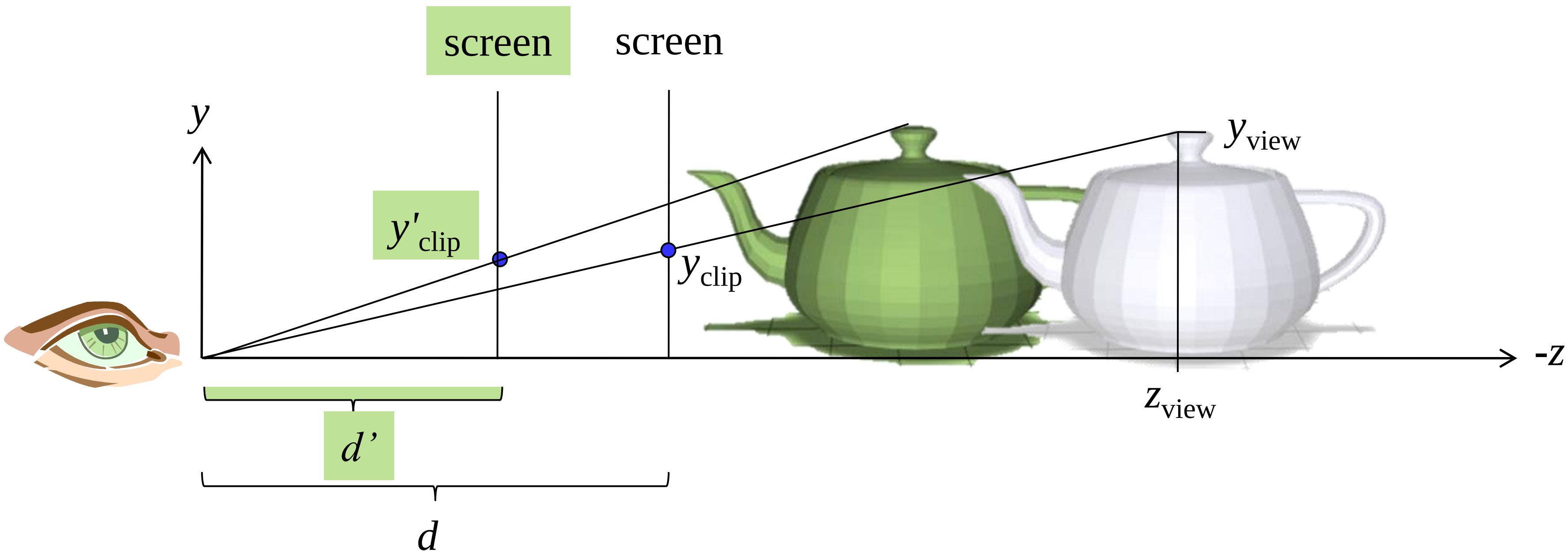
$y_{clip} = d\ y_{view}/(-z_{view})$

$y'_{clip} = d'\ y_{view}/(-z_{view}) = (d'/d)\ y_{clip}$

Changing parameter d just changes scale of projection



Parameter d

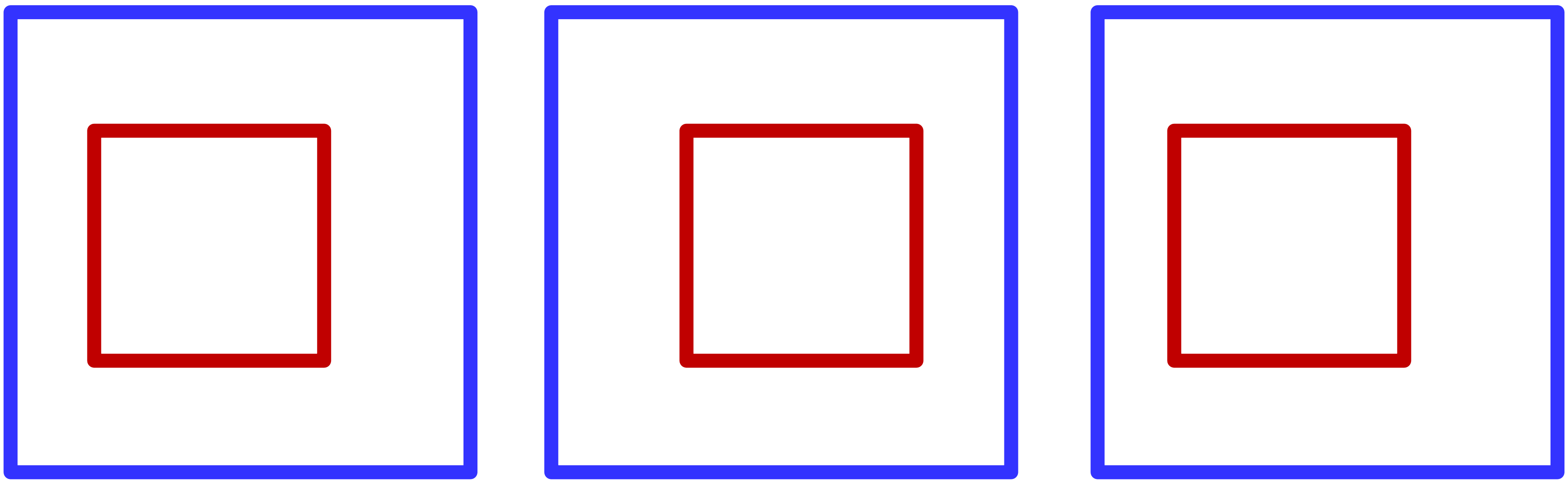


To change degree of perpective distortion, need to change distance from eye to scene,

…by moving scene closer or farther to eye,

… along z axis in viewing coordinates

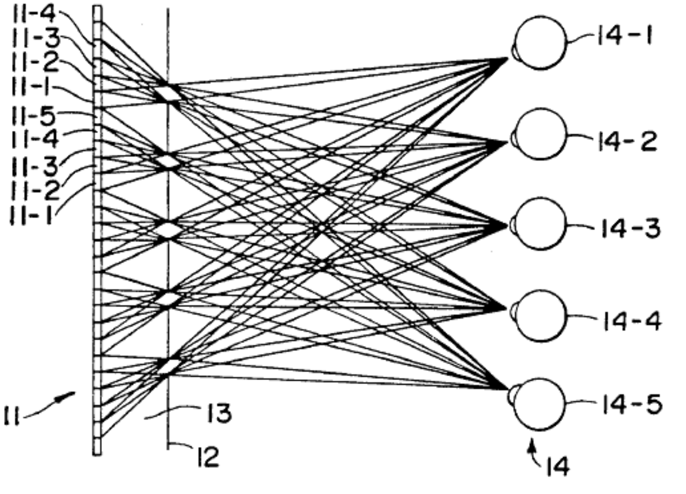
Stereo



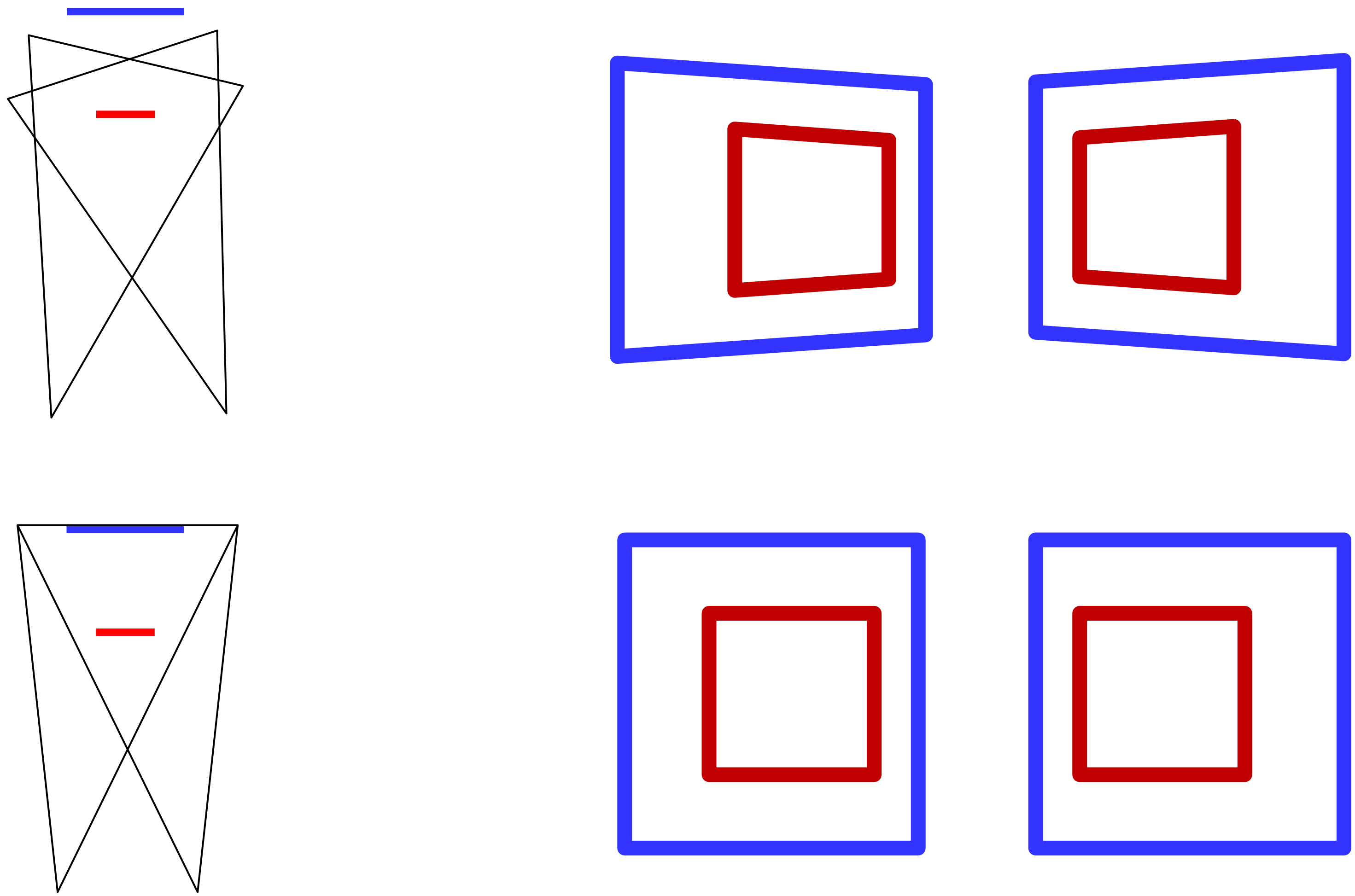



Stereo

- Disparity – differences (in image distance) between similar features images (varies with depth)
- Stereo methods
  - Cross eye & wall eye
  - Anaglyph (colored glasses)
  - Polarized glasses
  - Field sequential using alternately blinking lcd's in the glasses
  - Autostereograms (barrier strip or lenticular)



Rotation v. Shear



Sheared Perspective

- Shear first, then perspective
- Shear should preserve plane distance *f* from eyepoint
- Shear should move eyepoint *d* units perp to view direction
- Translate +*f* in *z* direction (remember view in -*z* dir)
- Shear the point (0,0,*f*) to the point (-*d*,0,*f*) *(opposite shear)*
- Translate back, by (0,0,-*f*)
- Apply perspective